to a very thin slot in the center conductor of a microstrip line were evaluated. These results should be divided by 4 because, in the calculation of ΔV, the effect of loading has to be taken into consideration.

The real part of the normalized impedance was found to be negligibly small (0.0020 at 3 GHz). The reactance as a function of frequency is plotted in Fig. 3. However, this approach is applicable only in the case of a very thin slot.

C. The Present Method

The reflection coefficient due to the normalized series reactance (jX_b) is given by

$$\Gamma = Z_n/(2+Z_n) = jX_b/(2+jX_b). \quad (21)$$

When loading due to the discontinuity is small, the reactance can be approximated as

$$jX_b \approx 2\Gamma \quad (22)$$

where Γ is the reflection coefficient computed from (16).

The series reactance was computed for a narrow transverse slot of width 1 mm and length 1.5 cm in a 20 Ω line on an RT Duroid substrate of height 1/16 in. The variation of the normalized series reactance, X_b, is shown in Fig. 3. The series reactance calculated from Oliner's formula (18) using the $\boldsymbol{e}$ and $\boldsymbol{h}$ in microstrip are also shown in the figure for comparison. The results for the case of the circular aperture of diameter 1.5 cm are shown in Fig. 4.

IV. Experimental Verification

In order to verify the theoretical results, a thin slot of width 1 mm and length 1.5 cm was etched in a microstrip line having a characteristic impedance of 20 Ω and a width of 1.58 cm on an RT Duroid substrate of height 1/16 in. The use of a low-impedance line with this substrate was essential. The width of a 50 Ω line is 4.71 mm, and a slot less than 4.7 mm long in this line cannot produce appreciable loading at the input. Thus, the measure of complex reflection and transmission coefficients is extremely difficult in 50 Ω line. For appreciable loading, it was necessary to etch the discontinuity in a low-impedance line which was matched both at the input and output to a standard 50 Ω line with the help of a taper as depicted in Fig. 1.

In order to evaluate the effect of the discontinuity alone, a similar line was fabricated without the discontinuity. The return loss caused by the taper alone was better than 20 dB, showing excellent match. The complex transmission coefficient, T, was measured with an HP sweep oscillator and an HP S-parameter test unit by sweeping the frequency from 0.5 to 2.0 GHz. The calibration was done at a few spot frequencies. In terms of complex transmission coefficient $T = |T|e^{j\theta}$, the required reactance is given by [1]

$$X = -2\sin\theta/|T|. \quad (23)$$

The measured data at spot frequencies for the slot and for a circular aperture of 1.5 cm diameter are shown along with the theoretical results in Fig. 3 and Fig. 4 respectively.

V. Conclusions

Closed-form expressions for the series reactance of two types of discontinuities in the center conductor of microstrip line have been developed. The results based on the quasi-static analysis using equivalent dipole moments are in good agreement with those obtained by modifying the formulation of Oliner [7] and Das [2] and with the experimental data from transmission measurements. Knowledge of the equivalent network parameter is essential in the analysis of periodic structures using such discontinuities.

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Analysis of Stripline Filled with Multiple Dielectric Regions

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Abstract—**The determination of the characteristic impedance of the stripline filled with different dielectric regions is discussed in this paper. Four rectangular dielectric regions whose interfaces are perpendicular to the ground planes are considered. The data on the characteristic impedance and effective dielectric constant are presented for the case of stripline filled with four rectangular regions having different dielectric constants. Results for impedance are compared with the data in the literature. Design data (w/b ratio and effective dielectric constant) are also presented.**

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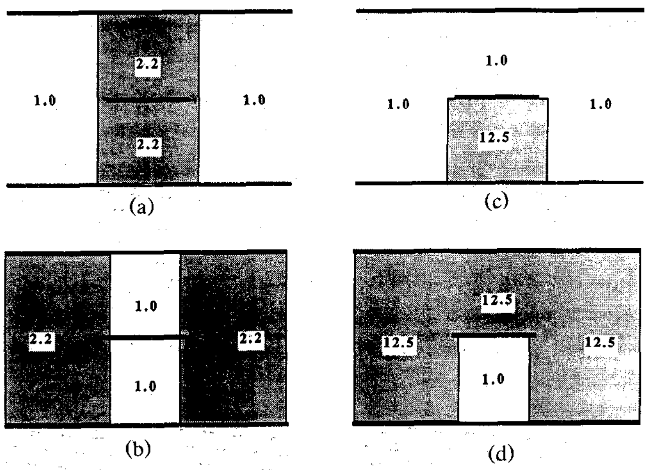

Fig. 1. Two stripline configurations (a and c) and their complementary structures (b and d), each filled with four different rectangular dielectric regions.

I. Introduction

There are a few published investigations of nonuniformly filled strip and microstrip lines [1]–[3]. Studies have also been made on structures filled with anisotropic multilayered dielectric media [4], [5]. So far no simple formulation is available on the impedance evaluation of a stripline with multiple dielectric regions having the interfaces perpendicular to the ground planes. A knowledge of the impedance of a stripline with mixed dielectric regions is useful in evaluating the overall performance of a stripline circuit mounted with active devices. These studies are also useful for transmission-type permittivity measurements of complex dielectric materials such as liquids, solids, and tissues. By knowing the change in the overall transmission coefficient with and without the test dielectric material, the permittivity of the material can be determined.

Hence, a symmetric stripline structure filled with different rectangular dielectric regions with their interfaces perpendicular to the ground planes is analyzed using conformal transformation. With the application of conformal transformation, the stripline structure is transformed into a parallel-plate configuration and the interfaces between the dielectric regions in the plane of the structure are transformed to curved lines. The parallel-plate configuration of the stripline structure contains two types of dielectric regions, mixed and uniformly filled. The capacitance due to the mixed dielectric region depends on the respective relative dielectric constants and the curved boundary separating the two regions. The capacitance due to the mixed dielectric portion of the parallel-plate capacitor is evaluated by dividing the region into a large number of columns with extremely small column widths and considering the parallel combination of the capacitances due to each column. The capacitance due to each column is further determined by the series combination of the capacitances due to each dielectric region considering the respective heights of the dielectric columns and relative dielectric constants. Finally, the characteristic impedance and the effective dielectric constant are determined from a knowledge of the total capacitance. Keeping the present printed technology in mind, four structures, which are indicated in Fig. 1, are chosen for this analysis. However, the present analysis does not pose any restriction on the number of dielectric regions.

The results on the characteristic impedance and the effective dielectric constant for the four structures of the stripline are presented for different values of dielectric constants (Fig. 1). The results for structure (c) of Fig. 1 are compared with those of

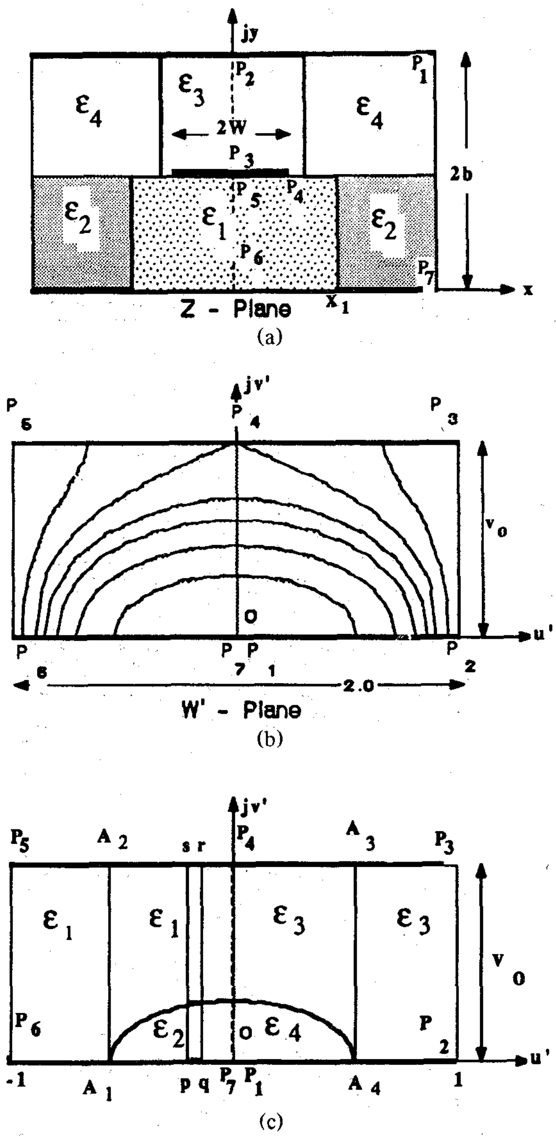

Fig. 2. Stripline filled with four rectangular dielectric regions and its conformal representation.

Joshi and Das [2], in which case the relative dielectric constant was 12.5. Design data for a 50 Ω line of normalized center conductor width and corresponding effective dielectric constant are presented as a function of the width of the rectangular dielectric region for all four structures of Fig. 1.

II. Formulation

The basic configuration of stripline filled with four different rectangular dielectric regions with interfaces perpendicular to the ground planes is shown in Fig. 2(a). Using the conformal transformation, one half of the structure, ($P_1, P_2, \cdots, P_6$ and P_7) of Fig. 2(a) (z plane), is mapped into a rectangle in two stages as shown in Fig. 2(b) (w' plane). The equations for the above transformations are available in the literature [7]. However, these equations are given here in the Appendix for completeness. The interface between two rectangular regions is conformally mapped to a curved line as shown in Fig. 2(b) [6]. Since the interface between the two dielectric regions is perpendicular to the ground planes, after transformation, it intersects the real axis at the bottom and top plates of the parallel-plate configuration of the stripline. However, the intersection of the interface with the top plate is conditional. The interfaces lying beyond the edge of the center strip in the z plane will intersect the boundary between the left and right portions of the parallel-plate capacitor in the transformed plane (w' plane). As the position of the interface changes from $x = 0$ to $x = \infty$ in the z plane, the corresponding shape and boundary changes from $u' = -1$ to 0 for y are less than b, and from $u' = 1$ to 0 for y are greater than b in the w' plane.

The following steps describe the estimation of the capacitance of the parallel-plate configuration of the stripline, which is shown in Fig. 2(c). The curved boundary in this figure corresponds to the case where the interface in the z plane is away from the center conductor, i.e., $x_1 > w$. The total capacitance, C, of the parallel-plate configuration of Fig. 2(c) is obtained by considering the parallel combination of C_l and C_r, where C_l and C_r are the capacitances arising from the rectangles $P_7P_4P_5P_6$ and $P_1P_2P_3P_4$, respectively. Further, the capacitances C_l and C_r can be obtained by considering the parallel combination of C_f and C_v. C_f is the capacitance of the uniformly filled dielectric region due to the rectangle $A_1A_2P_5P_6$ for C_l and $P_2P_3A_3A_4$ for C_r. C_v is the capacitance of the mixed dielectric region due to the rectangles $A_1A_2P_4P_7$ for C_l and $A_4A_3P_4P_1$ for C_r.

The expression for the capacitance of the mixed dielectric region due to the portion $A_1A_2P_4P_7$ can be derived by dividing it into a number of columns, each having incremental width $\Delta u_l'$ and height v_o, and summing over the interval 0 to $-u_l'$. The quantity u_l' is the distance between the points P_7 and A_1 along the u' axis in the w' plane. The incremental capacitance ΔC_l of the column $pqrs$ (Fig. 2(c)) is obtained by considering a series combination of the capacitances due to the two dielectric layers and is given by

$$\Delta C_l = \frac{\Delta u_l' \epsilon_0}{\dfrac{v_l'}{\epsilon_2} + \dfrac{(v_o - v_l')}{\epsilon_1}} \tag{1}$$

where v_l' is the position of the interface between the first and second dielectric blocks in the w' plane, and ϵ_1 and ϵ_2 are the corresponding relative dielectric constants. By partitioning the mixed dielectric region into a large number of columns the boundary between the two dielectric regions in each column may be assumed to be parallel to the plates. The capacitance thus estimated has some error. To minimize the error one lets $\Delta u_l' \to 0$ in the above equation. The number of columns then approaches infinity. The expression for the capacitance of the mixed dielectric region reduces to

$$C_v = \int_0^{-u_l'} \frac{du_l' \epsilon_0}{\dfrac{v_l'}{\epsilon_2} + \dfrac{(v_o - v_l')}{\epsilon_1}}. \tag{2}$$

The capacitance of the uniformly filled region, C_f, due to the portion $A_1A_2P_5P_6$ is given by

$$C_f = \frac{\epsilon_1(1 - u_l')}{v_o}. \tag{3}$$

Using (2) and (3), the capacitance C_l can be expressed as

$$C_l = C_f + C_v. \tag{4}$$

By following the above procedure the expression for the capacitance C_r can be written as

$$C_r = \frac{\epsilon_3(1 - u_r')}{v_o} + \int_0^{u_r'} \frac{du_l' \epsilon_0}{\dfrac{v_l'}{\epsilon_4} + \dfrac{(v_o - v_l')}{\epsilon_3}} \tag{5}$$

where u_r' is the distance between points A_4 and P_2 along the u' axis in the w' plane (Fig. 2(c)). The integrations appearing in the expressions for the capacitances C_r and C_l are numerically evaluated by an adaptive quadrature method [8]. The procedure for evaluating the expressions of (2)–(4) and (5) is described below. The interface coordinates in the z plane and the dimensions of the stripline are necessary for this procedure.

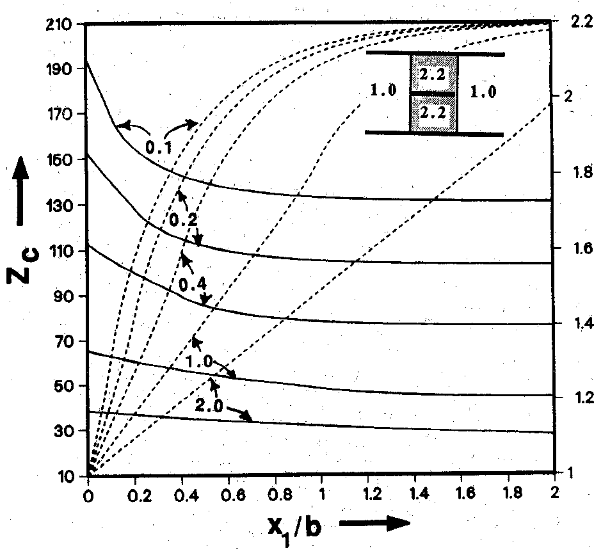

Fig. 3. Variation of characteristic impedance and effective dielectric constant as a function of the width of the rectangular dielectric region along the x direction for strip widths 0.1, 0.2, 0.4, 1.0, and 2.0 (case (a) of Fig. 1): —— Z_c; ---- ϵ_{eff}.

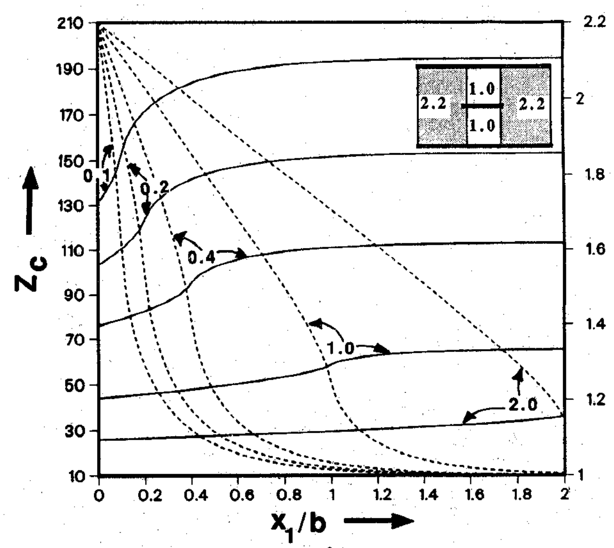

Fig. 4. Variation of characteristic impedance and effective dielectric constant as a function of the width of the rectangular dielectric region along the x direction for strip widths 0.1, 0.2, 0.4, 1.0, and 2.0 (case (b) of Fig. 1): —— Z_c; ---- ϵ_{eff}.

1) Determine the value of m for the given w/b ratio using (A6).

2a) Determine the intersection of the dielectric interface with the bottom plate of the parallel-plate configuration of the stripline (Fig. 2(c)) i.e., $-u_{l1}'$:
   i) For $\gamma = \pi/2$, determine the value of β satisfying the condition $x_1 = \text{Real}(z)$ using (A1) and (A4).
   ii) Calculate $-u_{l1}'$ using (A2) and (A3).

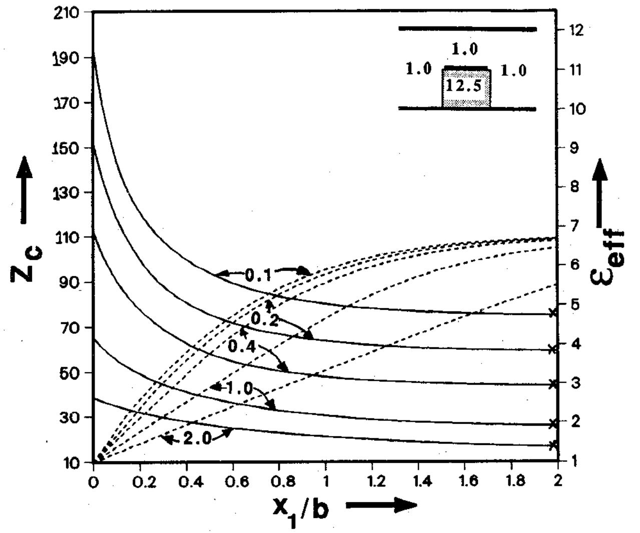

Fig. 5. Variation of characteristic impedance and effective dielectric constant as a function of the width of the rectangular dielectric region along the x direction for strip widths 0.1, 0.2, 0.4, 1.0, and 2.0 (case (c) of Fig. 1): ——— Z_c; ---- ϵ_{eff}; × × × Joshi and Das [2] for $\epsilon_r = 12.5$ and $x_1/b = 2.0$.

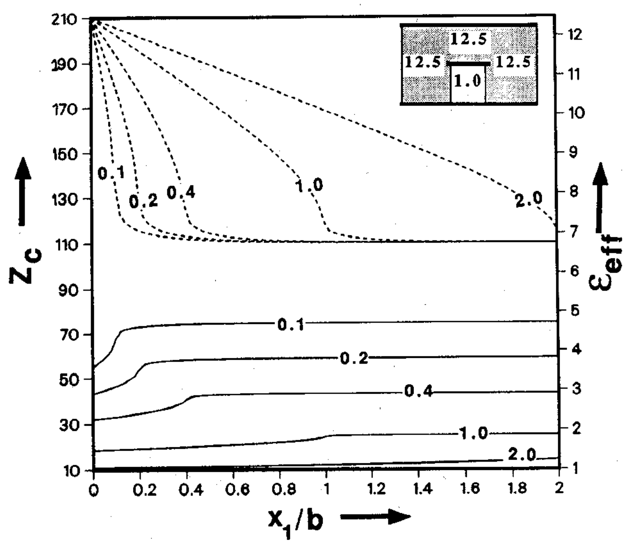

Fig. 6. Variation of characteristic impedance and effective dielectric constant as a function of the width of the rectangular dielectric region along the x direction for strip widths 0.1, 0.2, 0.4, 1.0, and 2.0 (case (d) of Fig. 1): ——— Z_c; ---- ϵ_{eff}.

2b) If $x_1 < w$ in the z plane there exists another intersection of the interface with the top plate of the parallel-plate capacitor of the stripline in the w' plane. It can be determined as in step 2a, except for the case $\gamma = 0$. The intersection with the line $v_i' = v_o$ occurs for $u_i' = u_{i2}'$. For $x_1 \geqslant w$, $u_{i2}' = 0$.

3) Evaluate the expression given in (2) numerically using the following procedure:
   i) Select a u_i' value within the range $-u_{i1}'$ to $-u_{i2}'$.
   ii) Determine the value of β for the selected u_i' using (A5).
   iii) Find γ by solving the transcendental equation $x_1 = \text{Real}(z)$.
   iv) Determine v' from (A2) and (A3).
   v) Calculate the value of the integrand given in (2).
   vi) Increment u_i' by a small value and go to step (ii).

TABLE I

w/b	x_1/w		
	1.0	1.5	2.0
0.1	29.506	22.455	18.176
0.2	24.326	16.124	11.524
0.4	17.795	09.072	04.998
1.0	09.494	02.186	00.508
2.0	05.294	00.000	00.000

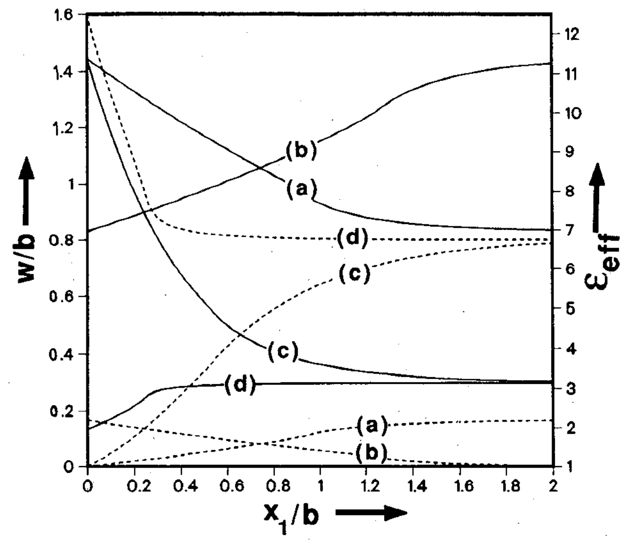

Fig. 7. Variation of strip width and the corresponding value of effective dielectric constant as a function of the width of the rectangular dielectric region along the x direction for a 50 Ω line impedance for the configurations shown in Fig. 1: ——— w/b; ---- ϵ_{eff}.

4) Calculate the capacitance C_l using (2)–(4).
5) Similarly, calculate the value of the capacitance C_r using (5).

Since the stripline structure is symmetric about the y axis, the total capacitance, C, is twice that of the capacitance of the parallel-plate configuration of Fig. 2(c) and is calculated from

$$C = 2(C_l + C_r). \tag{6}$$

From the value of the capacitance C thus obtained, the expression for the characteristic impedance is obtained from

$$Z_c = 30\pi v_o \sqrt{\frac{C_o}{C}} = \frac{Z_o}{\sqrt{\epsilon_{\text{eff}}}} \tag{7}$$

where $C_o = 4\epsilon_o / v_o$, $v_o = K'(m)/K(m)$ (K being a complete elliptic integral of the first kind with the given argument and modulus), and ϵ_{eff} is the effective dielectric constant.

The variations of the characteristic impedance and effective dielectric constant as a function of the width of the rectangular dielectric region (x_1/b) are calculated for the four cases given in Fig. 1 and are shown in Figs. 3–6. In practice, striplines are fabricated with a finite-width substrate. Using the configuration given in Fig. 1(a) the effect of the finite dielectric substrate width on the characteristic impedance is estimated. Table I shows the percentage deviation in the characteristic impedance as a function of the ratio of dielectric filling to strip width, i.e., x_1/w. The results for various values of w/b (for the configuration Fig. 1(a)) are given. The percent deviation is defined as

$$\%\ \text{deviation} = 100\frac{(Z_p - Z_f)}{Z_f} \tag{8}$$

where Z_f is the characteristic impedance of the uniformly filled stripline (infinite substrate), and Z_p is the characteristic impedance of a partially filled stripline (finite-width substrate with infinite ground plane). Finally the design data for a 50 Ω line for the four configurations shown in Fig. 1 are determined. Results on the w/b ratio and on the effective dielectric constant as a function of x_1/b are presented in Fig. 7.

III. Results and Discussion

The analysis presented in this paper describes a method for determining the characteristic impedance of the stripline structure when filled with rectangular dielectric regions with their interfaces perpendicular to the ground planes. By the simple application of conformal transformation, it is possible to estimate the characteristic impedance and effective dielectric constant with considerable reduction in the computer time compared with other numerical methods (including the quasi-static approximation). The results presented in Fig. 3 are for the configuration of Fig. 1(a), where the relative dielectric constant is 2.2. It can be observed that for small values of w/b the characteristic impedance of inhomogeneously filled stripline approaches the impedance value of completely filled stripline for large values of x_1/b. However, for large values of w/b, this limit is approached for smaller values of x_1/b. Further, a wide strip has a less significant fringing field. From the data presented in Table I, it is evident that for w/b greater than 0.4, the percentage error in the characteristic impedance is less than 9% (the x_1/w ratio is 1.5). The data presented in Table I are quite useful in fabricating stripline circuits. Using these data, the designer can have quantitative information about the deviation from the designed characteristic impedance value based on an infinite substrate assumption. Further, one can also start the design stage with the present method (finite-width substrate).

From the results shown in Figs. 3 to 6, it can be observed that the characteristic impedance changes most significantly when $x_1/b = w/b$. This could be due to the fact that the characteristic impedance is extremely sensitive to the filling of the dielectric around the center strip because of the discontinuity of charge at the edges of the center strip. This phenomenon is more prominent for the cases when the region below and above the center strip is filled with a low relative dielectric constant and the remaining region is filled with a higher relative dielectric constant (Figs. 4 and 6). The variation of characteristic impedance as well as the effective dielectric constant is nearly linear with x_1/b except in the region around $x_1/b = w/b$. The characteristic impedance data presented in Figs. 4–6 are useful for active device mounting where the center strip is supported on one or two dielectric columns. It can be seen from these results that for very large values of w/b ($w/b \geqslant 1.0$), the impedance varies linearly with x_1/b for x_1/b less than w/b. For larger w/b ratios, the difference in the characteristic impedance with and without dielectric filling around the center strip is small. This fact can be utilized for mounting active devices in strip transmission lines while calculating w/b ratios for lower impedance values. The characteristic impedance for the construction in Fig. 1(c) with $x_1/b = 2.0$, $x_1/b = \infty$, and $\epsilon_r = 12.5$ is compared with the results calculated using the analysis of Joshi and Das [2]. This comparison is indicated in Fig. 5. It is observed that the difference between the line impedances for $x_1/b = 2.0$ and $x_1/b = \infty$ is extremely small. The variation of effective dielectric constant is also computed for the two pairs of complementary dielectric filling as a function of w/b (Figs. 3–6). These data can be used to calculate the w/b ratios for the specified impedance values.

In Fig. 7, the w/b ratio for a characteristic impedance value of 50 Ω is presented as a function of x_1/b ratio. From this figure, it is seen that the effective dielectric constant and the w/b ratio vary linearly with x_1/b in two distinct regions. This information is also valuable in estimating the change in the characteristic impedance and effective dielectric constant caused by the variations in stripline dimensions.

Appendix

The conformal transformation of one half of the stripline (z plane) to the upper half of the t plane is given as [7]

$$z = x + jy = \frac{2b}{\pi} \ln\left[\frac{\sqrt{(1-mt^2)} + \sqrt{(m-mt^2)}}{\sqrt{(1-m)}}\right] + jb \quad \text{(A1)}$$

where $2b$ is the thickness of the stripline structure and $2w$ is the width of the center strip of stripline. The upper half of the t plane is transformed into a rectangle, shown in Fig. 2(b) (w' plane), and is given by

$$w' = u' + jv' = \frac{-F(\phi|m)}{K(m)} + j\frac{K'(m)}{K(m)} \quad \text{(A2)}$$

where m is a constant, and F and K correspond to incomplete and complete elliptic integrals of the first kind, respectively. The incomplete elliptic integral $F(\phi|m)$ with complex argument can be expressed in terms of two incomplete elliptic integrals with real arguments:

$$F(\phi|m) = F(\beta|m) + jF(\gamma|m_1) \quad \text{(A3)}$$

where $m_1 = (1-m)$, $\phi = \sin^{-1} t$, and $t = t_r + jt_i$, with

$$t_r = \frac{\sin\beta\sqrt{(1-m_1\sin^2\gamma)}}{\cos^2\gamma + m\sin^2\beta\sin^2\gamma} \quad \text{(A4a)}$$

$$t_i = \frac{\cos\beta\cos\gamma\sin\gamma\sqrt{(1-m\sin^2\beta)}}{\cos^2\gamma + m\sin^2\beta\sin^2\gamma}. \quad \text{(A4b)}$$

Substituting (A3) in (A2) and solving for β, we have

$$\beta = am(-u'K(m)). \quad \text{(A5)}$$

The expression of strip width in terms of m can be found from (A1) by substituting x and y values, i.e., $x = w$ and $y = b$, and is given as

$$\frac{w}{b} = \frac{1}{\pi}\ln\left[\frac{1+\sqrt{m}}{1-\sqrt{m}}\right]. \quad \text{(A6)}$$

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